

USITP-94-02

hep-ph/9402325

QCD corrections to direct $B \rightarrow J/\Psi$ decays

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Abstract

We calculate next to leading order QCD corrections to the direct decays $b \rightarrow J/\Psi + X$ and $b \rightarrow \eta_c + X$. The strong renormalization scale dependence is seen to persist also in this order and in fact the rate is driven to an unphysical negative value at the M_b scale. We show that this is a consequence of the strong suppression and scale dependence in the leading order term. Large cancellations take place between terms from three different orders in α_s . Lacking a third order calculation we are forced to throw away the leading order term and all but one of the next to leading order terms. The remaining $c\bar{c}$ gluon emission term could very well be the dominant one. Even if this is not the case, the picture of a rate at least as suppressed as in the leading logarithmic approximation survives.

Recently, there has been renewed interest in the theoretical treatment of various decays involving B mesons (see e.g. [1, 2, 3, 4]). Since it now appears likely that dedicated B “factories” may be built in the near future, it is of great importance to match the new level of experimental accuracy to state-of-the-art calculations within the Standard Model. Only then can one gain sensitivity to eventual new physics in the study of B decays. Even within the Standard Model, one has to make sure that the strong interaction physics involved when relating the Kobayashi-Maskawa origin of CP violation to measured B decays is well enough understood to establish this final piece of the six-quark Standard Model.

Presently, most of the B decay phenomenology seems to be reasonably well understood, with some notable exceptions. First, the measured semileptonic branching ratio of B mesons seems to be too low (i.e., the non-leptonic branching ratio seems to be too high) compared to theoretical calculations. This has already led the authors of [4] to put the question whether there is new, exotic physics involved in B decays. Secondly, the QCD-improved effective non-leptonic Hamiltonian seems to give a too small a branching ratio of $B \rightarrow J/\Psi + X$, even if cascade decays are taken into account [1].

Both the semileptonic and nonleptonic decay rates have been calculated in the next to leading logarithmic approximation. In the calculation of the next to leading order correction to the nonleptonic rate, quark masses were put to zero. After phase space integration this leads to a complete cancellation of one of three next to leading order terms (each with different Wilson coefficient structure). It has been argued that neither higher order QCD corrections nor nonperturbative corrections are large enough to explain the low semileptonic branching ratio [4].

In the case of the $B \rightarrow J/\Psi + X$ decay, a next to leading order calculation of the perturbative QCD corrections has been lacking (in a previous attempt [5] the renormalization group was not treated properly [1]).

In this paper, we present the results of a calculation of $B \rightarrow J/\Psi + X$ in the next to leading logarithmic approximation (for details, see [6]). In a straightforward way we also extend this to η_c production.

As will be seen, the reason that the rate for direct $B \rightarrow J/\Psi + X$ decay comes out small is that the Wilson coefficient of the colour singlet effective four fermion operator is strongly suppressed by the QCD evolution and becomes zero just below the M_b scale. In the next to leading order, colour octet contributions from $c\bar{c}$ bremsstrahlung appear. These are not suppressed by

the QCD evolution and could therefore be as important as the leading order term. This is obscured in the next to leading order calculation by the strong scale dependence of the one-loop contributions, making the rate negative at the M_b scale. We show that this is a consequence of the strong suppression and scale dependence of the colour singlet Wilson coefficient, and propose a slight modification of the resummation to overcome this problem. After a careful analysis, the next to leading order calculation is seen to indicate a further suppression of the rate as compared to the leading order at the M_b scale. This reinforces the suspicion that the bulk of J/Ψ s produced in B decays originate from other mechanisms.

The nonleptonic effective Hamiltonian describing the B decays we treat here can be written [2, 3]

$$\begin{aligned}
\mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_1(\mu) O_1 + C_2(\mu) O_2) \\
&= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_+(\mu) O_+ + C_-(\mu) O_-) \\
&= \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left(\frac{1}{N} C_0(\mu) O_1 + 2C_2(\mu) O_8 \right). \tag{1}
\end{aligned}$$

Here V are Cabibbo-Kobayashi-Maskawa flavour mixing matrix elements (we neglect the Cabibbo suppressed channel $c \rightarrow u$ compared to $c \rightarrow s$), G_F is the Fermi weak coupling constant and N is the number of colours (three for QCD). The O 's are four fermion shortdistance operators, all with the same $(V - A) \otimes (V - A)$ γ matrix structure but with different colour structures, while the $C(\mu)$'s are the corresponding renormalization scale dependent Wilson coefficient functions. In the last line of Eq. (1) the effective Hamiltonian is written as a linear combination of a colour singlet (O_1) and a colour octet (O_8) operator, which is the most relevant form for our process. The operators O_{\pm} are introduced for convenience since they do not mix under evolution. We give explicitly

$$\begin{aligned}
O_1 &= (\bar{b}_\alpha c_\beta)_{V-A} \otimes (\bar{c}_\beta s_\alpha)_{V-A} \\
O_2 &= (\bar{b}_\alpha c_\alpha)_{V-A} \otimes (\bar{c}_\beta s_\beta)_{V-A} \\
O_8 &= \frac{1}{2} O_2 - \frac{1}{2N} O_1
\end{aligned}$$

$$O_{\pm} = (O_2 \pm O_1)/2. \quad (2)$$

Eqs. (1) and (2) lead to the following relations for the Wilson coefficients:

$$\begin{aligned} C_0(\mu) &= \frac{N+1}{2}C_+(\mu) - \frac{N-1}{2}C_-(\mu) \\ C_2(\mu) &= (C_+(\mu) + C_-(\mu))/2. \end{aligned} \quad (3)$$

In the leading logarithmic approximation, the C_{\pm} were calculated long ago [7, 8], with the result

$$L_{\pm}(\mu) \equiv C_{\pm}^0 = \left(\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right)^{d_{\pm}}, \quad (4)$$

where the anomalous dimensions d_{\pm} are given by

$$d_{\pm} = \frac{\gamma_{\pm}^{(0)}}{2\beta_0}, \quad (5)$$

with

$$\gamma_{\pm}^{(0)} = \frac{\pm 12(N \mp 1)}{2N} \quad (6)$$

and

$$\beta_0 = \frac{11}{3}N - \frac{2}{3}n_F, \quad (7)$$

where n_F is the number of active flavours at the scale μ .

This leads to the decay rate for $B \rightarrow J/\Psi + X$ [9]

$$\Gamma_{B \rightarrow J/\psi + X}^0 = L_0^2 G_0, \quad (8)$$

where L_0 and L_2 are related to L_{\pm} in the same way as C_0 and C_2 are related to C_{\pm} (eq. 3), and with

$$G_0 = \mathcal{K}(1-x)^2(1+2x), \quad (9)$$

where

$$\mathcal{K} = \frac{G_F^2 |V_{cb}|^2 |V_{cs}|^2 M_b^3 R_s(0)^2}{96 \pi^2 M_c} \quad (10)$$

$$x = \frac{4M_c^2}{M_b^2}. \quad (11)$$

Here the nonperturbative coupling of the $c\bar{c}$ pair to the bound state is parametrized by the S wave function at the origin, $R_s(0)$, which can be measured (including order α_s corrections) in J/Ψ decays to lepton pairs.

Due to the strong suppression of L_0 in the expression for Γ_0 (Eq. (8)), the lowest order width is very small, and strongly dependent on the scale parameter μ (Fig. 1(a),2). In fact, with the choice $\mu \sim 2.3 \text{ GeV}$, it can even be made to vanish. The hope has been that the next to leading order corrections would both increase the rate and make it less sensitive to μ . In next to leading order, several new contributions to the Wilson coefficient functions appear, summarized by the expression

$$C_{\pm}(\mu) = L_{\pm}(\mu) \left(1 + \frac{\alpha_s(\mu)}{4\pi} B_{\pm} \right) \left(1 + \frac{\alpha_s(M_W) - \alpha_s(\mu)}{4\pi} R_{\pm} \right). \quad (12)$$

We expect the contribution from penguin induced operators to be small for our process (for a calculation of their size for the general $\Delta B = 1$ Hamiltonian, see [11, 12]). Note also that for colour singlet $c\bar{c}$ production penguin graphs do not appear until second order in α_s . We work in the 't Hooft-Veltman regularization scheme [10], where B_{\pm} and R_{\pm} are related by

$$R_{\pm} = B_{\pm} + \frac{(N \mp 1)}{4N\beta_0} (-21 \pm 57/N \pm 23N \mp 4n_F) - (d_{\pm}/\beta_0)/\beta_1, \quad (13)$$

(see [3] for a more complete discussion, including the effects of altering the regularization scheme) The beta function coefficient β_1 is given by

$$\beta_1 = \frac{34}{3}N^2 - \frac{10}{2}Nn_f - \frac{(N^2 - 1)}{N}n_F. \quad (14)$$

We find, in agreement with [3]

$$B_{\pm} = \frac{\pm B(N \mp 1)}{2N}, \quad (15)$$

where $B = 7$ in the 't Hooft-Veltman regularization scheme. After a calculation of one-loop and bremsstrahlung contributions we find the rate for $b \rightarrow J/\Psi + X$ in the next to leading logarithmic approximation,

$$\begin{aligned} \Gamma_{B \rightarrow J/\psi + X} &= L_0^2 G_0 + \frac{\alpha_s(M_W) - \alpha_s(\mu)}{4\pi} C_F G_0 \times \\ &\times \left(2L_0 L_2 (R_+ - R_-) + L_0^2 \left(\frac{2R_+}{N-1} + \frac{2R_-}{N+1} \right) \right) \\ &+ \frac{\alpha_s(\mu)}{4\pi} C_F \left(L_2^2 G_3 - L_0 L_2 (G_2 - 2B G_0) - L_0^2 G_1 \right). \end{aligned} \quad (16)$$

Here $C_F = (N^2 - 1)/2N$. G_3 stems from real gluon emission, whereas G_1 and G_2 are a mixture of real and virtual contributions. These functions G_i , $i = 1, 2, 3$ are given by

$$\begin{aligned} G_1 &= \mathcal{K} \left(2(1-x)^2 (5+4x) \log(1-x) + \right. \\ &\quad \left. 4x(1-2x)(1+x) \log(x) + (1-2x)(1-x)(3+5x) \right) + \\ &\quad G_0 \left(4\pi^2/3 + 4 \log(1-x) \log(x) + 8 \mathcal{L}i_2(x) \right) \end{aligned} \quad (17)$$

$$\begin{aligned} G_2 &= \mathcal{K} \left(\frac{2(1-x)(34+23x-51x^2+16x^3)}{2-x} + \right. \\ &\quad \frac{32(1-x)^3 x}{2-x} \log(2) - \frac{8(1-x)^3(3-x^2)}{(2-x)^2} \log(1-x) + \\ &\quad \left. \frac{4x^2(26-19x+4x^2)}{2-x} \log(x) \right) + G_0 \left(12 \log\left(\frac{\mu^2}{M_b^2}\right) - 4 \right) \end{aligned} \quad (18)$$

$$G_3 = \mathcal{K} \left(\frac{4}{9} (1-x) (1+37x-8x^2) - \frac{8}{3} (1-6x) \log(x) \right), \quad (19)$$

where $\mathcal{L}i_2(x)$ is the dilogarithmic function

$$\mathcal{L}i_2(x) = - \int_0^x dt \frac{\log(1-t)}{t}. \quad (20)$$

An important term to notice in (18) is the scale dependent term proportional to $12 G_0(x) \log(\mu^2/M_b^2)$ in G_2 .

The main difference between our result in Eq. (16) and that of [5] lies in the treatment of the large logarithmic corrections that come from the box

diagrams, and the way they are summed using the renormalization group. In [5] the J/Ψ particle was treated as being a fundamental, colour singlet field produced by the leading logarithmic effective Hamiltonian, to which gluonic corrections were applied. This does not seem to be correct, since all short distance interactions that may eventually give rise to a J/Ψ should be added coherently. There could possibly be other production mechanisms, e.g., a $(v/c)^2$ suppressed (but not L_0 suppressed) soft gluon induced fragmentation process. This does not, however, effect the calculation of the hard subprocess rates. Only after the formation time $\sim 1/M_\Psi$ can the J/Ψ particle be considered to be a fundamental, colour singlet state. Indeed, as pointed out in [1], the renormalization group summation of the leading logarithmic terms in [5] does not reproduce the $O(\alpha_s^2)$ perturbative result correctly. Already at the outset of the calculation in [5] cancellations are performed between terms that in a correct treatment should be multiplied by different combinations of Wilson coefficient functions. It is therefore impossible to reconstruct the correct result from this calculation. Going the opposite way, trying to reproduce numerical values given in [5] for some classes of diagrams from our calculations, we still find some minor discrepancies.

In Fig. 1 (curves (a) and (b)) we show the leading and our next to leading order result for $B \rightarrow J/\Psi + X$. As can be seen, the next to leading order corrections are very large and the result still depends dramatically on the scale μ . For large values of μ the rate is even driven to unphysical negative values, suggesting that higher order corrections are still anomalously large.

To understand this behaviour we look at the expansion of the leading order term $G_0 L_0^2(\mu^2)$ around some scale μ^* close to the zero point of L_0

$$\begin{aligned} L_0^2(\mu)G_0 &= L_0^2(\mu^*)G_0 + \frac{\alpha_s(\mu^*)C_F}{4\pi}12G_0L_0(\mu^*)L_2(\mu^*)\log(\mu^2/\mu^{*2}) + \\ &\quad \left(\frac{\alpha_s(\mu^*)C_F}{4\pi}\right)^2 \left(36G_0L_2^2(\mu^*)\log^2(\mu^2/\mu^{*2}) + \mathcal{O}(L_0(\mu^*))\right) \end{aligned} \quad (21)$$

We note that the renormalization scale dependence is dominated by the second order term in this expansion. In the next to leading order (16), the first order term in (21) is cancelled by the μ dependent term in G_2 (18):

$$\begin{aligned}
& - \frac{\alpha_s(\mu)C_F}{4\pi} 12G_0L_0(\mu)L_2(\mu) \log(\mu^2/\mu^{*2}) = \\
& - \frac{\alpha_s(\mu^*)C_F}{4\pi} 12G_0L_0(\mu^*)L_2(\mu^*) \log(\mu^2/\mu^{*2}) - \\
& \left(\frac{\alpha_s(\mu^*)C_F}{4\pi} \right)^2 (72G_0L_2^2(\mu^*) \log^2(\mu^2/\mu^{*2}) + \mathcal{O}(L_0(\mu^*))) \quad (22)
\end{aligned}$$

After the cancellation only a second order μ dependent term remains. This term is twice as large as the one dominating the renormalization scale dependence of the leading order term (21), however, and moreover has the opposite sign.

To conclude, the L_0^2 suppression postpones the cancellation of the leading order renormalization scale dependence until the second order. The next to leading order corrections merely mirror the leading order scale dependence. Thus, if the appropriate scale for the process is close to the zero point of L_0 (curve (a) in Fig. 1) the qualitative picture of the scale dependence in the next to leading logarithmic approximation (curve (b) in Fig. 1) is an inevitable consequence of the strong scale dependence and suppression in the leading logarithmic approximation.

To overcome this problem we propose a simultaneous expansion in L_0 and α_s . This amounts to a rather modest resummation, mixing terms from three orders in α_s . In this way terms proportional to L_0 or L_0^2 are added together with the terms that cancel their extreme scale dependence. Since we have already seen that we do not gain any information by adding these terms separately we do not lose anything by doing the expansion this way. Let us now analyze if we can gain something.

In the new L_0 - α_s expansion we find that the old leading term $L_0^2G_0$ is replaced by the square of the two bremsstrahlung diagrams from the $c\bar{c}$ pair — the G_3 term in Eq. (16). We note that the G_3 term dominates $L_0^2G_0$ only at scales close to the zero point of L_0 . From the argument above we know, however, that the extreme scale dependence of $L_0^2G_0$ will be cancelled in the new scheme, and it is only after this cancellation that we can expect to get something numerically small, as compared to the G_3 term. The correct question to ask is obviously - will the next to leading order term in the L_0 - α_s expansion be numerically small?

In a complete next to leading order calculation in the new scheme one is forced to calculate all α_s^2 terms that are not L_0 suppressed. Such a calculation could possibly be performed since all corrections coming from two-loop diagrams or from higher order matching and evolution of the effective weak theory are L_0 suppressed.

We have already calculated several of the terms needed in such a new next to leading order calculation. Lacking a complete calculation we try to get as much information as possible from the calculated terms and from the structure of the unknown terms. The missing terms are all proportional to $\alpha_s^2 L_2^2$ and are therefore α_s suppressed as compared to the leading order term. We know, however, that they contain the term $36G_0 \log^2(\mu^2)$ that cancels the scale dependences in the G_0 and G_2 terms (21,22). It is the large numerical prefactor of this \log^2 term that threatens the viability of the expansion.

The G_2 term in eq. (16) is dominated by the cross term between a group of one-loop diagrams and the born graph. It is the square of this group of diagrams that contain the dangerous \log^2 term. Since the UV-divergences of the one loop diagrams have the same gamma matrix structure as the born diagram, we can use $G_2^2/(4G_0)$ as a rough estimate of the square of the one-loop diagrams. If we add this approximate term to the next to leading order terms that we have calculated exactly, we get

$$\begin{aligned}
\Gamma_{B \rightarrow J/\psi + X} &\approx \frac{\alpha_s(\mu)}{4\pi} C_F G_3 L_2^2 \times \\
&\left(1 - \frac{\alpha_s(\mu)}{4\pi} C_F \frac{2B}{N} + \frac{\Delta\alpha_s}{4\pi} C_F \left(\frac{2R_+}{N+1} + \frac{2R_-}{N-1} \right) \right) + \\
&G_0 \left(L_0^2 + \frac{\Delta\alpha_s}{4\pi} C_F L_0 L_2 2R_1 + \left(\frac{\Delta\alpha_s}{4\pi} \right)^2 C_F^2 L_2^2 R_1^2 \right) - \\
&\frac{\alpha_s(\mu)}{4\pi} C_F (G_2 - 2BG_0) \left(L_0 L_2 + \frac{\Delta\alpha_s}{4\pi} C_F L_2^2 R_1 \right) + \\
&\left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 C_F^2 L_2^2 \frac{(G_2 - 2BG_0)^2}{4G_0}, \tag{23}
\end{aligned}$$

where $R_1 \equiv R_+ - R_-$ and $\Delta\alpha_s \equiv (\alpha_s(M_W) - \alpha_s(\mu))$.

Numerically we find that the cancellation is quite effective (Fig. 1 (c)). Remember that the born term, which is several times larger than the new

leading order term at the M_b scale, is one of the terms in the correction. This incomplete second order calculation should not be viewed as an approximation to the complete one. It is only meant to show that the cancellation between the terms which are the largest, viewed separately, does take place. We have completely neglected the double bremsstrahlung and one-loop bremsstrahlung diagrams appearing at the $\alpha_s^2 L_2^2$ level. The next to leading order correction in the L_0 - α_s expansion could very well be substantial, but there is reason to believe that it is at least not larger than the leading term. In Fig. 2 we display the scale dependence of the relevant ingredients in our scheme L_0 , L_2 and α_s separately.

In the numerical calculations we used $\Lambda_{QCD}^{(5)} = 190$ MeV ($\alpha_s(M_Z) = 0.115$), $M_b = 5$ GeV and $x \equiv 4M_c^2/M_b^2 = (M_\psi/M_{B^0})^2 \approx 0.38$.

Varying Λ_{QCD} and M_b (with M_c/M_b fixed) we find - not surprisingly - that the L_0 - α_s expansion works best for large α_s and low M_b when the zero point of L_0 is close to the M_b scale. With $\alpha_s(M_Z) = 0.120$ the cancellation is almost perfect while it is already beginning to break down at $\alpha_s(M_Z) = 0.110$. Furthermore, we note that the hard gluon emission term G_3 is more sensitive to variations in M_c/M_b or M_s/M_b (see [6] for expressions to first order in M_s^2/M_b^2) than the other terms. Thus a 5% lower value for M_c/M_b results in a 20% increase in the leading order term in the L_0 - α_s expansion. If we increase M_s/M_b from 0 to 0.1 corresponding to $M_s \approx 0.5$ GeV we find a 5% suppression of G_0 while G_3 is suppressed by substantial 17%.

We have calculated next to leading order corrections also for η_c production in B-meson decays, and find an almost identical pattern as in the case of J/ψ production. To check the scheme dependence of our calculations, we have also utilized the naive dimensional regularization (NDR) scheme, with negligible differences in the final results.

To be able to compare our results with experiments we first have to deal with the strong overall dependence on heavy quark masses, the Kobayashi-Maskawa matrix element V_{cb} and on $R_S(0)$, the value of the radial wave function for the $c\bar{c}$ pair at zero distance. Using the measured rate (5.36 ± 0.29 keV) for the electromagnetic decay of the J/ψ [13] and the next to leading order result

$$\Gamma_{J/\psi \rightarrow e^+e^-} = \frac{16\alpha_E^2 |R_S(0)|^2}{9M_\psi^2} \left(1 - \frac{16\alpha_s}{3\pi}\right) \quad (24)$$

we find $|R_S(0)|^2/M_c^2 \approx 0.5$ GeV. This must, however, be considered a very

rough estimate since the QCD corrections in (24) are extremely large. In the leading order we would get $|R_S(0)|^2 / M_c^2 \approx 0.2 \text{ GeV}$. Hopefully lattice calculations will soon give more precise estimates. To cancel the V_{cb} dependence and to reduce the very strong M_b dependence we normalize our results to the semileptonic branching ratio. To next to leading order in α_s the semileptonic decay rate is [2]

$$\Gamma_{SL} = \frac{G_F^2 |V_{cb}|^2}{192 \pi^3} M_b^5 g(M_c/M_b) \left(1 - \frac{2\alpha_s(\mu)}{3\pi} f(M_c/M_b) \right) \quad (25)$$

where

$$g(x) = 1 - 8x^2 - 24x^4 \log(x) + 8x^6 - x^8 \quad (26)$$

and $f(x)$ can be found in tabulated form in [14]. Using $M_c/M_b = M_{D^0}/M_{B^0} \approx 0.35$ in (25) with α_s at 5 GeV and the measured semileptonic branching ratio $(10.7 \pm 0.5)\%$ [13] we find

$$\begin{aligned} Br_{B \rightarrow J/\psi + X} &\equiv \frac{\Gamma_{B \rightarrow J/\psi + X}}{\Gamma_{tot}} = \left(\frac{Br_{SL} G_0}{\Gamma_{SL}} \right) \frac{\Gamma_{B \rightarrow J/\psi + X}}{G_0} \\ &\approx (0.2\%) \times \left(\frac{5 \text{ GeV}}{M_b} \right) \left(\frac{|R_S(0)|^2 / M_c^2}{0.5 \text{ GeV}} \right) \left(\frac{\Gamma_{B \rightarrow J/\psi + X} / G_0}{0.05} \right) \end{aligned} \quad (27)$$

The measured inclusive branching fraction for ψ production in B decays is $(1.12 \pm 0.16)\%$. Using measured branching fractions for B decays into ψ' , χ_{c1} and for the cascade decays $\psi' \rightarrow \psi + X$, $\psi' \rightarrow \chi_{c1} + \gamma$ and $\chi_{c1} \rightarrow \psi + \gamma$, one finds the branching fraction (0.71 ± 0.20) [1] for direct J/ψ production in B decays, assuming that no other cascade decays give significant contributions to the J/ψ production. Using the large value 0.5 GeV for $|R_S(0)|^2 / M_c^2$ and taking the size of the QCD-suppression to be ~ 0.05 from Fig. 1, we are still two and a half standard deviations from the experimental result.

We thus conclude from our analysis that unless the experimental value goes down as more data accumulate, it seems that the decay $B \rightarrow J/\Psi + X$ is a process that cannot presently be well described by a standard application of perturbative QCD to zeroth order in the relative velocity of the $c\bar{c}$ quarks. We have pointed out that the strong QCD suppression of the colour singlet operator give rise to strong cancellations between terms appearing in different orders in α_s . Although we have suggested a method to ameliorate this problem, there is room for other QCD effects, subleading in most other

cases, that could bridge the gap between the predicted and observed values for the branching ratio. It is thus not at all necessary at this point to make the conclusion that exotic mechanisms (i.e. beyond the Standard Model) are needed to explain the large experimental rate for this type of decay.

We thank S. Åminneborg and R. Robinett for useful discussions. This work was supported by the Swedish Natural Science Research Council.

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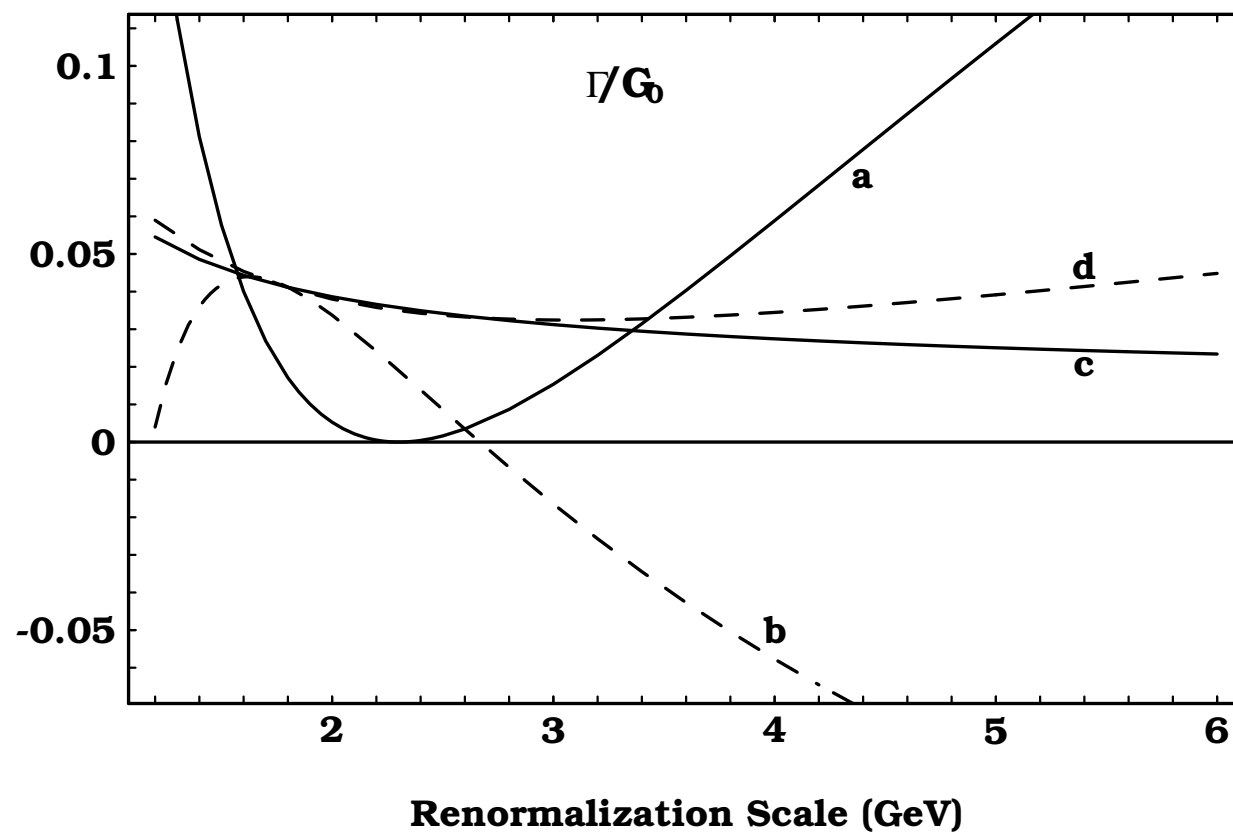
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Figure Captions

Fig. 1. Renormalization scale dependence of $\Gamma_{B \rightarrow J/\Psi + X}$ normalized to the naive parton model result G_0 , in various approximations:

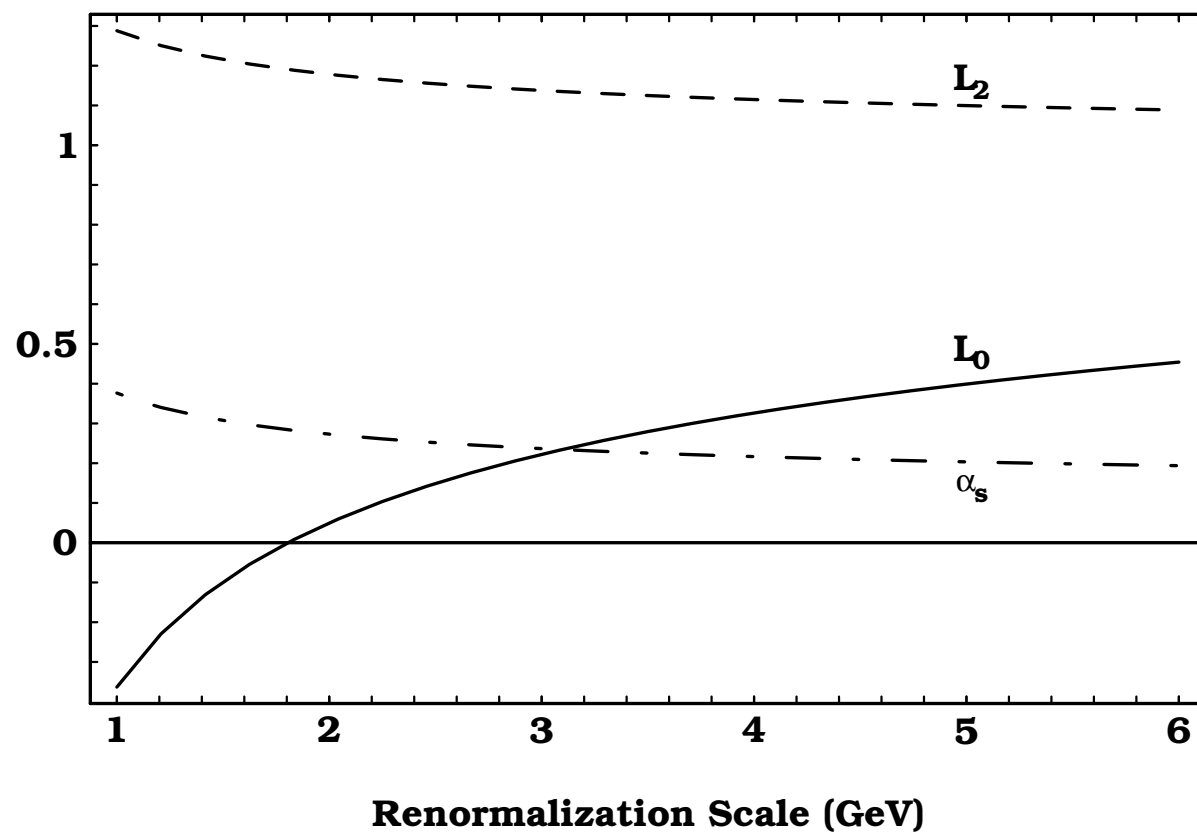
- (a) Leading logarithmic approximation (for consistency, the leading order result for α_s is used in the definition of L_{\pm}).
- (b) Conventional next to leading logarithmic approximation.
- (c) Leading order result in the proposed L_0 - α_s expansion.
- (d) Incomplete next to leading order result in the L_0 - α_s expansion, as given by Eq. (23).

Fig. 2. Renormalization scale dependence of the colour singlet and octet Wilson coefficient functions L_0 , L_2 and of α_s .



This figure "fig1-1.png" is available in "png" format from:

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